

# Calculating Tank Volume

Saving time, increasing accuracy

By Dan Jones, Ph.D., P.E.

Calculating fluid volume in a horizontal or vertical cylindrical or elliptical tank can be complicated, depending on fluid height and the shape of the heads (ends) of a horizontal tank or the bottom of a vertical tank. Exact equations now are available for several commonly encountered tank shapes. These equations can be used to make rapid and accurate fluid-volume calculations. All equations are rigorous, but computational difficulties will arise in certain limiting configurations.

All volume equations give fluid volumes in cubic units from tank dimensions in consistent linear units. All variables defining tank shapes required for tank volume calculations are defined in the "Variables and Definitions" sidebar. Graphically, Figs. 1 and 2 show horizontal tank variables and Figs. 3 and 4 show vertical tank variables.

Exact fluid volumes in elliptical horizontal or vertical tanks can be found by first calculating the fluid volumes of appropriate cylindrical horizontal or vertical tanks using the equations described above, and then by adjusting those results using appropriate correction formulas.

## Horizontal Cylindrical Tanks

Fluid volume as a function of fluid height can be calculated for a horizontal cylindrical tank with either conical, ellipsoidal, guppy, spherical, or torispherical heads where the fluid height,  $h$ , is measured from the tank bottom to the fluid surface, see Figs. 1 and 2. A guppy head is a conical head where the apex of the conical head is level with the top of the cylindrical section of the tank as shown in Fig. 1. A torispherical head is an ASME-type head defined by a knuckle-radius parameter,  $k$ , and a dish-radius parameter,  $f$ , as shown in Fig. 2.

An ellipsoidal head must be exactly half of an ellipsoid of revolution; only a hemiellipsoid is valid – no "segment" of an ellipsoid will work as is true in the case of a spherical head where the head may be a spherical segment. For a spherical head,  $|a| \leq R$ , where  $R$  is the radius of the cylindrical tank body. Where concave conical, ellipsoidal, guppy, spherical, or torispherical heads are considered, then  $|a| \leq L/2$ .

Both heads of a horizontal cylindrical tank must be identical for the equations to work; i.e., if one head is conical, the other must be conical with the same dimensions. However, the equations can be combined to deal with fluid volume calculations of horizontal tanks with heads of different shapes. For instance, if a horizontal cylindrical tank has a conical head on one end and an ellipsoidal head on the other end, calculate fluid volumes of two tanks, one with conical heads and the other with ellipsoidal heads, and average the results to get the desired fluid volume. The heads of a horizontal tank may be flat ( $a = 0$ ), convex ( $a > 0$ ), or concave ( $a < 0$ ).

The following variables must be within the ranges stated:

- $|a| \leq R$  for spherical heads
- $|a| \leq L/2$  for concave ends
- $0 \leq h \leq 2R$  for all tanks
- $f > 0.5$  for torispherical heads
- $0 \leq k \leq 0.5$  for torispherical heads
- $D > 0$
- $L \geq 0$

**Variables and Definitions (See Figs. 1-5)**

**a** is the distance a horizontal tank's heads extend beyond ( $a > 0$ ) or into ( $a < 0$ ) its cylindrical section or the depth the bottom extends below the cylindrical section of a vertical tank. For a horizontal tank with flat heads or a vertical tank with a flat bottom  $a = 0$ .

**A<sub>f</sub>** is the cross-sectional area of the fluid in a horizontal tank's cylindrical section.

**D** is the diameter of the cylindrical section of a horizontal or vertical tank.

**D<sub>H</sub>, D<sub>W</sub>** are the height and width, respectively, of the ellipse defining the cross section of the body of a horizontal elliptical tank.

**D<sub>A</sub>, D<sub>B</sub>** are the major and minor axes, respectively, of the ellipse defining the cross section of the body of a vertical elliptical tank.

**f** is the dish-radius parameter for tanks with torispherical heads or bottoms;  $fD$  is the dish radius.

**h** is the height of fluid in a tank measured from the lowest part of the tank to the fluid surface.

**k** is the knuckle-radius parameter for tanks with torispherical heads or bottoms;  $kD$  is the knuckle radius.

**L** is the length of the cylindrical section of a horizontal tank.

**R** is the radius of the cylindrical section of a horizontal or vertical tank.

**r** is the radius of a spherical head for a horizontal tank or a spherical bottom of a vertical tank.

**V<sub>f</sub>** is the fluid volume, of fluid depth  $h$ , in a horizontal or vertical cylindrical tank.

### Horizontal Tank Equations

Here are the specific equations for fluid volumes in horizontal cylindrical tanks with conical, ellipsoidal, guppy, spherical, and torispherical heads (use radian angular measure for all trigonometric functions, and  $D/2 = R > 0$  for all equations):

*Conical heads.*

$$V_f = A_f L + \frac{2aR^2}{3} \times \begin{cases} K & \dots\dots\dots 0 \leq h < R \\ \pi/2 & \dots\dots\dots h = R \\ \pi - K & \dots\dots\dots R < h \leq 2R \end{cases}$$

$$K \equiv \cos^{-1} M + M^3 \cosh^{-1} \frac{1}{M} - 2M\sqrt{1 - M^2} \qquad M = \left| \frac{R - h}{R} \right|$$

*Ellipsoidal heads.*

$$V_f = A_f L + \pi a h^2 \left( 1 - \frac{h}{3R} \right)$$

*Guppy heads.*

$$V_f = A_f L + \frac{2aR^2}{3} \cos^{-1} \left( 1 - \frac{h}{R} \right) + \frac{2a}{9R} \sqrt{2Rh - h^2} (2h - 3R)(h + R)$$

*Spherical heads.*

$$V_f = A_f L + \begin{cases} \frac{\pi a}{6} (3R^2 + a^2) & \dots\dots\dots h = R, \quad |a| \leq R \\ \frac{\pi a}{3} (3R^2 + a^2) & \dots\dots\dots h = D, \quad |a| \leq R \\ \pi a h^2 \left( 1 - \frac{h}{3R} \right) & \dots\dots\dots h = 0 \text{ or } a = 0, R, -R \\ \frac{a}{|a|} \left[ \frac{2r^3}{3} \left[ \cos^{-1} \frac{R^2 - rw}{R(w-r)} + \cos^{-1} \frac{R^2 + rw}{R(w+r)} - \frac{z}{r} \left( 2 + \left( \frac{R}{r} \right)^2 \right) \cos^{-1} \frac{w}{R} \right] \right. \\ \qquad \qquad \qquad \left. - 2 \left( wr^2 - \frac{w^3}{3} \right) \tan^{-1} \frac{y}{z} + \frac{4wyz}{3} \right] & \dots\dots\dots h \neq R, D; \quad a \neq 0, R, -R; \quad |a| \geq 0.01D \\ \frac{a}{|a|} \left[ 2 \int_w^R (r^2 - x^2) \tan^{-1} \frac{\sqrt{R^2 - x^2}}{\sqrt{r^2 - R^2}} dx - A_f z \right] & \dots\dots\dots h \neq R, D; \quad a \neq 0, R, -R; \quad |a| < 0.01D \end{cases}$$

$$r = \frac{a^2 + R^2}{2|a|} \quad a \neq 0 \quad ; \quad a = \pm \left( r - \sqrt{r^2 - R^2} \right) \quad + (-) \text{ for convex (concave) heads}$$

$$w \equiv R - h \qquad y \equiv \sqrt{2Rh - h^2} \qquad z \equiv \sqrt{r^2 - R^2}$$

*Torispherical heads.*

In the  $V_f$  equation, use +(-) for convex(concave) heads.

$$V_f = A_f L \pm \begin{cases} 2 v_1 & \dots\dots\dots 0 \leq h \leq h_1 \\ 2 (v_{1,\max} + v_2 + v_3) & \dots\dots\dots h_1 < h < h_2 \\ 2 [2v_{1,\max} - v_1(h = D - h) + v_{2,\max} + v_{3,\max}] & \dots\dots\dots h_2 \leq h \leq D \end{cases}$$

$$v_1 \equiv \int_0^{\sqrt{2kDh-h^2}} \left[ n^2 \sin^{-1} \frac{\sqrt{n^2 - w^2}}{n} - w\sqrt{n^2 - w^2} \right] dx$$

$$v_2 \equiv \int_0^{kD \cos \alpha} \left[ n^2 \left( \cos^{-1} \frac{w}{n} - \cos^{-1} \frac{g}{n} \right) - w\sqrt{n^2 - w^2} + g\sqrt{n^2 - g^2} \right] dx$$

$$v_3 \equiv \begin{cases} \left[ \frac{r^3}{3} \left[ \cos^{-1} \frac{g^2 - rw}{g(w-r)} + \cos^{-1} \frac{g^2 + rw}{g(w+r)} - \frac{z}{r} \left( 2 + \left( \frac{g}{r} \right)^2 \right) \cos^{-1} \frac{w}{g} \right] - \left( wr^2 - \frac{w^3}{3} \right) \tan^{-1} \frac{\sqrt{g^2 - w^2}}{z} \right. \\ \quad \left. + \frac{wz\sqrt{g^2 - w^2}}{6} + \frac{wz}{2} \sqrt{2g(h-h_1) - (h-h_1)^2} \right] \dots\dots\dots 0.5 < f \leq 10 \\ \int_w^g (r^2 - x^2) \tan^{-1} \frac{\sqrt{g^2 - x^2}}{z} dx - \frac{z}{2} \left( g^2 \cos^{-1} \frac{w}{g} - w\sqrt{2g(h-h_1) - (h-h_1)^2} \right) \dots\dots 0.5 < f < 10,000 \end{cases}$$

$$v_{1,\max} \equiv v_1(h = h_1)$$

$$v_{2,\max} \equiv v_2(h = h_2)$$

$$v_{3,\max} \equiv v_3(h = h_2) = \frac{\pi a_1}{6} (3g^2 + a_1^2) \qquad a_1 \equiv r(1 - \cos \alpha)$$

$$\alpha \equiv \sin^{-1} \frac{1-2k}{2(f-k)} = \cos^{-1} \frac{\sqrt{4f^2 - 8fk + 4k - 1}}{2(f-k)} \qquad r \equiv fD$$

$$h_1 \equiv kD(1 - \sin \alpha)$$

$$h_2 \equiv D - h_1$$

$$n \equiv R - kD + \sqrt{k^2 D^2 - x^2}$$

$$w \equiv R - h$$

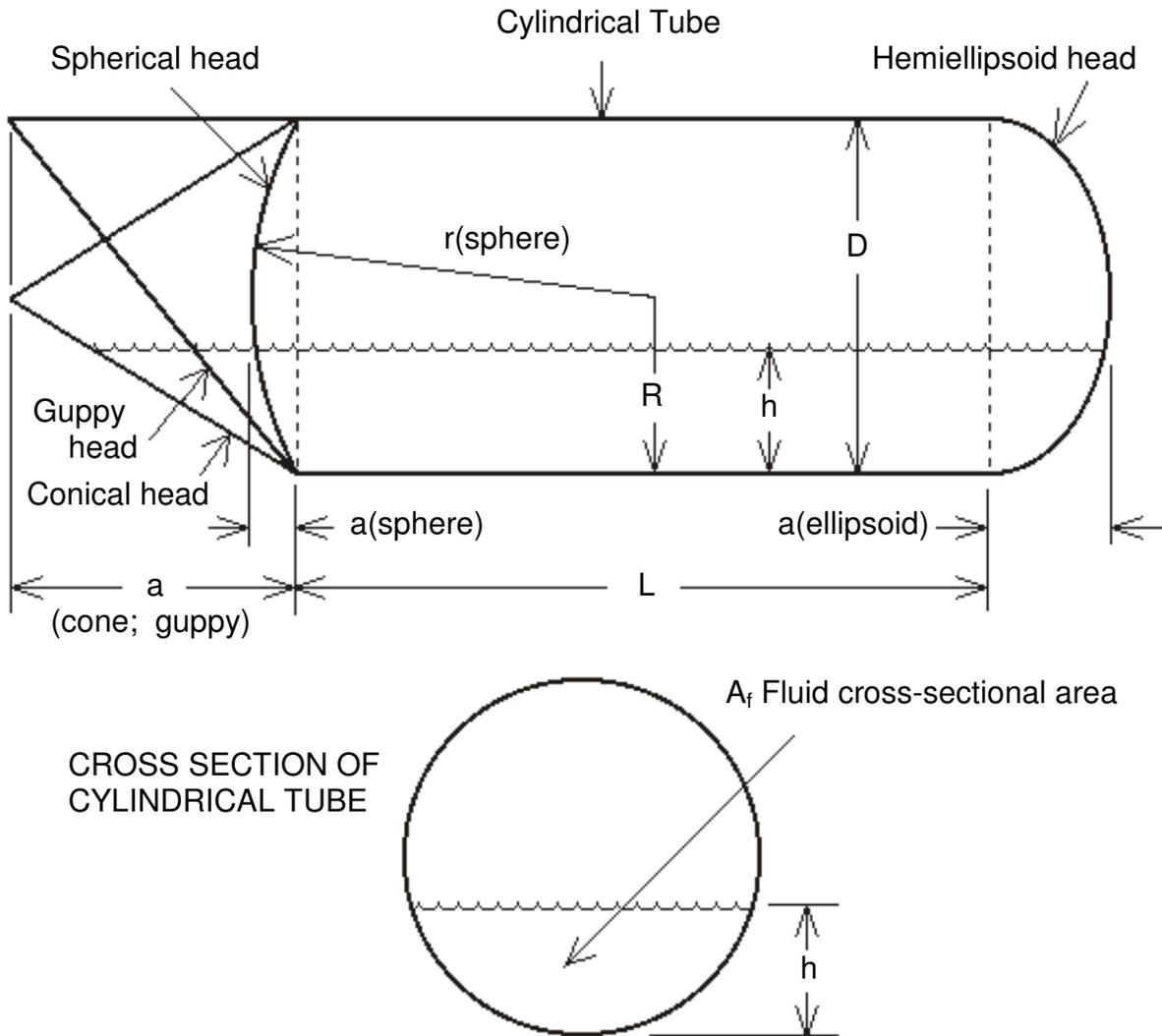
$$g \equiv fD \sin \alpha = r \sin \alpha$$

$$z \equiv \sqrt{r^2 - g^2} = fD \cos \alpha = r \cos \alpha$$

In the above equations,  $V_f$  is the total volume of fluid in the tank in cubic units consistent with the linear units of tank dimension parameters, and  $A_f$  is the cross-sectional area of fluid in the cylindrical body of the tank in square units consistent with the linear units used for  $R$  and  $h$ . The equation for  $A_f$  is given by:

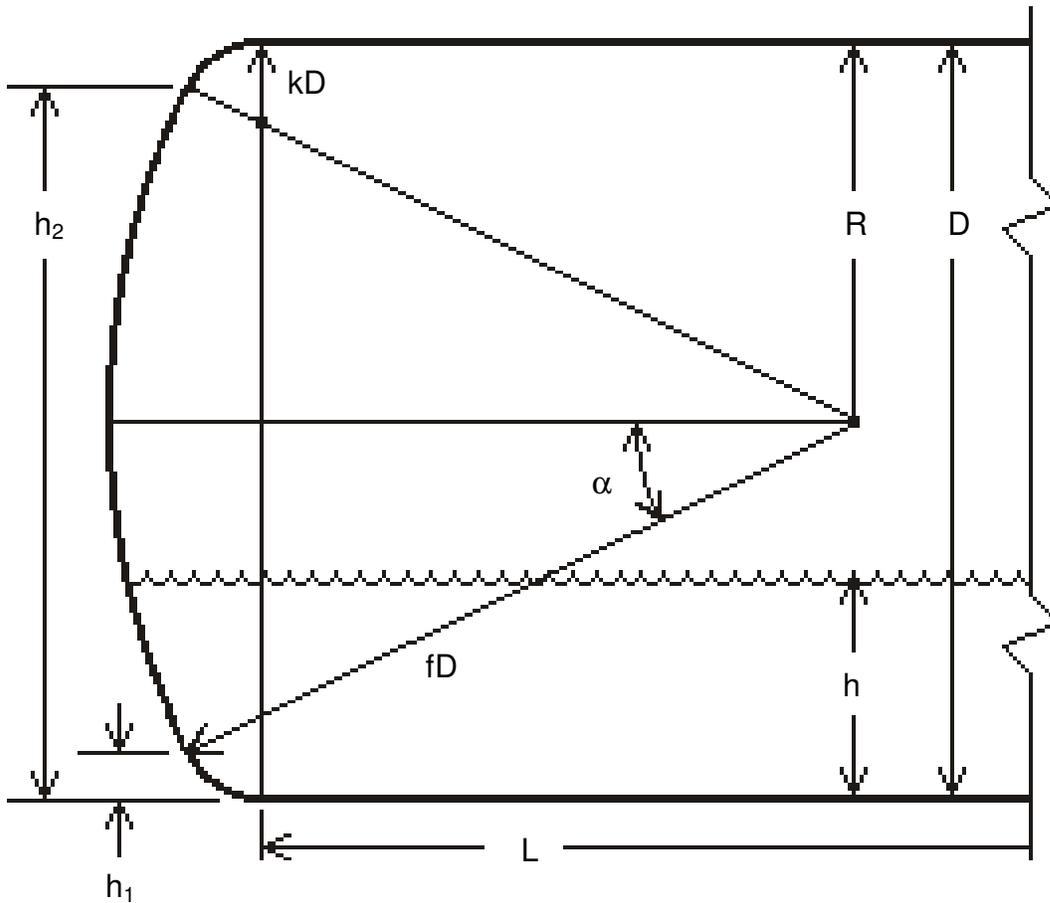
$$A_f = R^2 \cos^{-1} \left( \frac{R-h}{R} \right) - (R-h) \sqrt{2Rh - h^2}$$

**Figure 1. Parameters for Horizontal Cylindrical Tanks with Conical, Ellipsoidal, Guppy, or Spherical Heads.**



1. Both heads of a tank must be identical. Above diagram is for definition of parameters only.
2. Cylindrical tube of diameter  $D$  ( $D > 0$ ), radius  $R$  ( $R > 0$ ), and length  $L$  ( $L \geq 0$ ).
3. For spherical head of radius  $r$ ,  $r \geq R$  and  $|a| \leq R$ .
4. For convex head other than spherical,  $0 < a < \infty$ , for concave head  $a < 0$ .
5.  $L \geq 0$  for  $a \geq 0$ ,  $L \geq 2|a|$  for  $a < 0$ .
6. Ellipsoidal head must be exactly half of an ellipsoid of revolution.
7.  $0 \leq h \leq D$ .

**Figure 2. Parameters for Horizontal Cylindrical Tanks with Torispherical Heads.**



**Horizontal Cylindrical Tank Examples**

The following examples can be used to check application of the equations:

Find the volumes of fluid, in gallons, in horizontal cylindrical tanks 108" in diameter with cylinder lengths of 156", with conical, ellipsoidal, guppy, spherical, and "standard" ASME torispherical ( $f = 1$ ,  $k = 0.06$ ) heads, each head extending beyond the ends of the cylinder 42" (except torispherical), for fluid depths in the tanks of 36" (example 1) and 84" (example 2). Calculate five times for each fluid depth – for a conical, ellipsoidal, guppy, spherical, and torispherical head.

For example 1 the parameters are  $D = 108"$ ,  $L = 156"$ ,  $a = 42"$ ,  $h = 36"$ ,  $f = 1$ , and  $k = 0.06$ . The fluid volumes are 2,041.19 Gal for conical heads, 2,380.96 Gal for ellipsoidal heads, 1,931.72 Gal for guppy heads, 2,303.96 Gal for spherical heads, and 2,028.63 Gal for torispherical heads.

For example 2 the parameters are  $D = 108"$ ,  $L = 156"$ ,  $a = 42"$ ,  $h = 84"$ ,  $f = 1$ , and  $k = 0.06$ . The fluid volumes are 6,180.54 Gal for conical heads, 7,103.45 Gal for ellipsoidal heads, 5,954.11 Gal for guppy heads, 6,935.16 Gal for spherical heads, and 5,939.90 Gal for torispherical heads.

For torispherical heads, 'a' is not required input; it can be calculated from  $f$ ,  $k$ , and  $D$ . For these torispherical-head examples, the calculated value is 'a' = 18.288".

## Vertical Cylindrical Tanks

Fluid volume in a vertical cylindrical tank with either a conical, ellipsoidal, spherical, or torispherical bottom can be calculated, where the fluid height,  $h$ , is measured from the center of the bottom of the tank to the surface of the fluid in the tank. See Figs. 3 and 4 for tank configurations and dimension parameters, which are also defined in the “Variables and Definitions” sidebar.

A torispherical bottom is an ASME-type bottom defined by a knuckle-radius factor and a dish-radius factor as shown graphically in Fig. 4. The knuckle radius will then be  $kD$  and the dish radius will be  $fD$ . An ellipsoidal bottom must be exactly half of an ellipsoid of revolution. For a spherical bottom,  $|a| \leq R$ , where  $a$  is the depth of the spherical bottom and  $R$  is the radius of the cylindrical section of the tank.

The following parameter ranges must be observed:

- $a \geq 0$  for all vertical tanks,  $a \leq R$  for a spherical bottom
- $f > 0.5$  for a torispherical bottom
- $0 \leq k \leq 0.5$  for a torispherical bottom
- $D > 0$

## Vertical Tank Equations

Here are the specific equations for fluid volumes in vertical cylindrical tanks with conical, ellipsoidal, spherical, and torispherical bottoms (use radian angular measure for all trigonometric functions, and  $D > 0$  for all equations):

*Conical bottom.*

$$V_f = \begin{cases} \frac{\pi}{4} \left( \frac{Dh}{a} \right)^2 \left( \frac{h}{3} \right) & \dots\dots\dots h < a \\ \frac{\pi D^2}{4} \left( h - \frac{2a}{3} \right) & \dots\dots\dots h \geq a \end{cases}$$

*Ellipsoidal bottom.*

$$V_f = \begin{cases} \frac{\pi}{4} \left( \frac{Dh}{a} \right)^2 \left( a - \frac{h}{3} \right) & \dots\dots\dots h < a \\ \frac{\pi D^2}{4} \left( h - \frac{a}{3} \right) & \dots\dots\dots h \geq a \end{cases}$$

Spherical bottom.

$$V_f = \begin{cases} \frac{\pi h^2}{4} \left( 2a + \frac{D^2}{2a} - \frac{4h}{3} \right) & \dots\dots\dots h < a ; (a \leq D/2) \\ \frac{\pi}{4} \left( \frac{2a^3}{3} - \frac{aD^2}{2} + hD^2 \right) & \dots\dots\dots h \geq a ; (a \leq D/2) \end{cases}$$

Torispherical bottom.

$$V_f = \begin{cases} \frac{\pi h^2}{4} \left( 2a_1 + \frac{D_1^2}{2a_1} - \frac{4h}{3} \right) & \dots\dots\dots 0 \leq h \leq a_1 \\ \frac{\pi}{4} \left( \frac{2a_1^3}{3} + \frac{a_1 D_1^2}{2} \right) + \pi u \left[ \left( \frac{D}{2} - kD \right)^2 + s \right] + \frac{\pi t u^2}{2} - \frac{\pi u^3}{3} \\ \quad + \pi D(1-2k) \left[ \frac{2u-t}{4} \sqrt{s+tu-u^2} + \frac{t\sqrt{s}}{4} + \frac{k^2 D^2}{2} \left( \cos^{-1} \frac{t-2u}{2kD} - \alpha \right) \right] & \dots\dots a_1 < h \leq a_1 + a_2 \\ \frac{\pi}{4} \left( \frac{2a_1^3}{3} + \frac{a_1 D_1^2}{2} \right) + \frac{\pi t}{2} \left[ \left( \frac{D}{2} - kD \right)^2 + s \right] \\ \quad + \frac{\pi t^3}{12} + \pi D(1-2k) \left( \frac{t\sqrt{s}}{4} + \frac{k^2 D^2}{2} \sin^{-1}(\cos \alpha) \right) + \frac{\pi D^2}{4} [h - (a_1 + a_2)] & \dots\dots a_1 + a_2 < h \end{cases}$$

$$\alpha \equiv \sin^{-1} \frac{1-2k}{2(f-k)} = \cos^{-1} \frac{\sqrt{4f^2 - 8fk + 4k - 1}}{2(f-k)}$$

$$a_1 \equiv fD(1 - \cos \alpha)$$

$$a_2 \equiv kD \cos \alpha$$

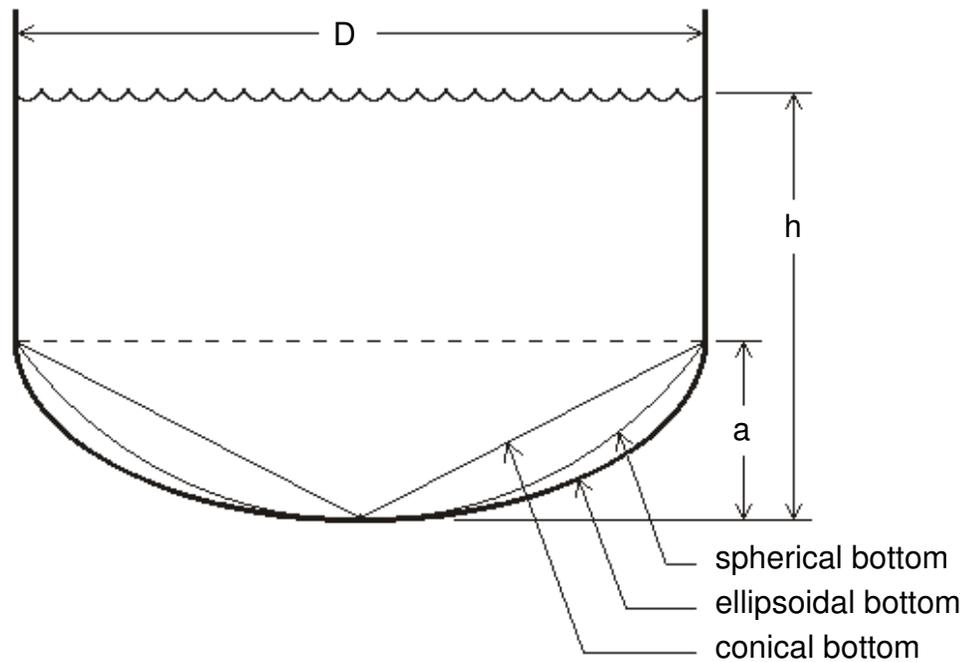
$$D_1 \equiv 2fD \sin \alpha$$

$$s \equiv (kD \sin \alpha)^2$$

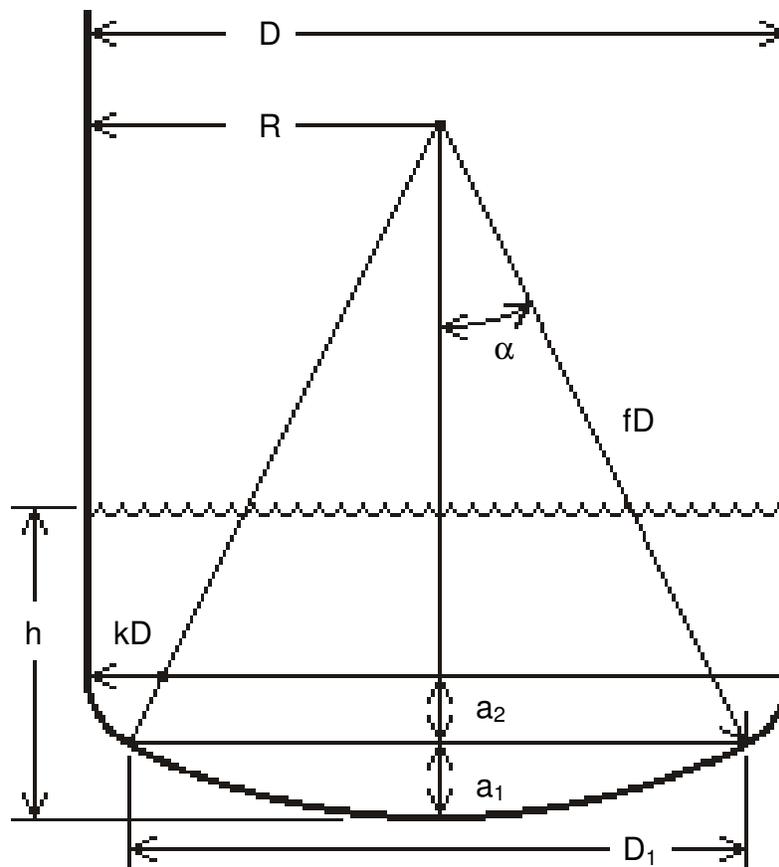
$$t \equiv 2kD \cos \alpha = 2a_2$$

$$u \equiv h - fD(1 - \cos \alpha)$$

**Figure 3. Parameters for Vertical Cylindrical Tanks with Conical, Ellipsoidal, or Spherical Bottoms.**



**Figure 4. Parameters for Vertical Cylindrical Tanks with Torispherical Bottoms.**



### Vertical Cylindrical Tank Examples

Two examples can be used to verify correct use of the equations for vertical cylindrical tanks; for each example calculate the fluid volumes for conical, ellipsoidal, spherical, and torispherical bottoms.

For example 1,  $D = 132"$ ,  $a = 33"$ ,  $h = 24"$ ,  $f = 1$ ,  $k = 0.06$ . The fluid volumes are 250.67 Gal for a conical bottom, 783.36 Gal for an ellipsoidal bottom, 583.60 Gal for a spherical bottom, and 904.07 Gal for a torispherical bottom.

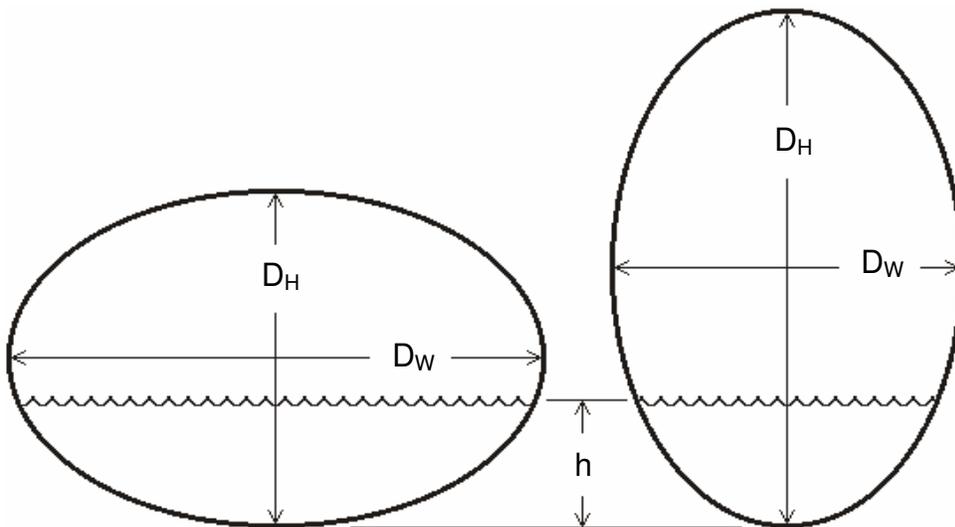
For example 2,  $D = 132"$ ,  $a = 33"$ ,  $h = 60"$ ,  $f = 1$ ,  $k = 0.06$ . The fluid volumes are 2,251.18 Gal for a conical bottom, 2,902.83 Gal for an ellipsoidal bottom, 2,658.46 Gal for a spherical bottom, and 3,036.76 Gal for a torispherical bottom.

In the case of a torispherical bottom, parameter 'a' is not required input, but can be calculated from the values of f, k, and D. In these examples, the calculated value is  $a = 22.353"$ .

### Horizontal and Vertical Elliptical Tanks

The previous sections dealt with horizontal and vertical tanks with cylindrical bodies, where the cross sections of the tank bodies are circles. This section deals with horizontal and vertical tanks with elliptical bodies, where the cross sections of the tank bodies are ellipses. For this article, a horizontal elliptical tank must be one of two possible configurations, shown in Fig. 5, where the major and minor axes of the elliptical cross sections are either vertical or horizontal.

**Figure 5. Cross Sections of Horizontal Elliptical Tanks.**



The heads of horizontal elliptical tanks and the bottoms of vertical elliptical tanks may be any of those described above for the corresponding cylindrical tanks with the assumption that the heads and bottoms are “deformed” proportionately to the deformation of the cylindrical body to form the elliptical body.

In certain cases, such as those with torispherical heads and bottoms and spherical heads and bottoms, it is necessary to distinguish which elliptical axis defines the head or bottom shape and which axis has been proportionately stretched or compressed from the cylindrical tank shape to form the elliptical tank shape; therefore, this distinction will be made for all cases for the sake of consistency, not necessity.

To calculate the fluid volume in a horizontal elliptical tank with the elliptical body oriented in one of the two orientations shown in Fig. 5, where the head parameters are defined in the vertical plane through the tank centerline (plane goes through  $D_H$ ), calculate the volume of a horizontal cylindrical tank with  $D = D_H$  using the equations above for horizontal cylindrical tanks with the appropriately-shaped heads. Multiply the volume found by  $D_W/D_H$  to get the desired elliptical tank fluid volume.

To calculate the fluid volume in a horizontal elliptical tank with the elliptical body oriented in one of the two orientations shown in Fig. 5, where the head parameters are defined in the horizontal plane through the tank centerline (plane goes through  $D_W$ ), calculate the volume of a horizontal cylindrical tank with  $D = D_W$  and a fluid height  $h' = h(D_W/D_H)$  using the equations above for horizontal cylindrical tanks with the appropriately-shaped heads. Multiply the volume found by  $D_H/D_W$  to get the desired elliptical tank fluid volume.

Examples for horizontal elliptical tanks: Find the fluid volumes in gallons of horizontal elliptical tanks with ellipsoidal, spherical, and torispherical heads with the following measurements:  $D_H = 100"$ ,  $D_W = 120"$ ,  $L = 156"$ ,  $a = 25"$  for ellipsoidal and spherical heads,  $f = 0.8$  and  $k = 0.1$  for torispherical heads, fluid heights  $h = 48"$ , head parameters of each tank defined (1) in a horizontal plane through the tank centerline and (2) in a vertical plane through the tank centerline. In Case 1, calculate horizontal cylindrical tank volumes with  $D = 120"$ ,  $L = 156"$ ,  $a = 25"$  for ellipsoidal and spherical heads,  $f = 0.8$  and  $k = 0.1$  for torispherical heads, and  $h = 57.6"$  ( $48" \times 120/100$ ) and multiply the volume found by  $100/120$ . In Case 2, calculate horizontal cylindrical tank volumes with  $D = 100"$ ,  $L = 156"$ ,  $a = 25"$  for ellipsoidal and spherical heads,  $f = 0.8$  and  $k = 0.1$  for torispherical heads, and  $h = 48$  and multiply the volume found by  $120/100$ . The results are summarized in the following table:

Case	Ellipsoidal heads (Gal)	Spherical heads (Gal)	Torispherical heads (Gal)
1	3,659.58	3,524.09	3,663.20
2	3,659.58	3,536.58	3,556.06

The values for 'a' in the torispherical-head cases above are 27.065" for Case 1 and 22.554" for Case 2.

For dealing with a vertical elliptical tank, define  $D_A$  and  $D_B$  to be the major and minor axes, respectively, of the ellipse defining the cross section of the tank body.

To calculate the fluid volume in a vertical elliptical tank, where the bottom parameters are defined in the plane through both the tank centerline and through  $D_A$ , calculate the volume of a vertical cylindrical tank with  $D = D_A$  using the equations above for a vertical cylindrical tank with the appropriately-shaped bottom. Multiply the volume found by  $D_B/D_A$  to get the desired elliptical tank fluid volume.

To calculate the fluid volume in a vertical elliptical tank, where the bottom parameters are defined in the plane through both the tank centerline and through  $D_B$ , calculate the volume of a vertical cylindrical tank with  $D = D_B$  using the equations above for a vertical cylindrical tank with the appropriately-shaped bottom. Multiply the volume found by  $D_A/D_B$  to get the desired elliptical tank fluid volume.

Examples for vertical elliptical tanks: Find the fluid volumes in gallons of vertical elliptical tanks with conical, spherical, and torispherical bottoms with the following measurements:  $D_A = 96"$ ,  $D_B = 72"$ ,  $a = 34"$  for conical and spherical bottoms,  $f = 0.9$  and  $k = 0.2$  for the torispherical bottom, fluid heights  $h = 53"$ , head parameters of each tank defined (1) in a plane through the tank centerline and  $D_A$  and (2) in a plane through the tank centerline and  $D_B$ . In Case 1, calculate vertical cylindrical tank volumes with  $D = 96"$ ,  $a = 34"$  (for conical and spherical bottoms),  $f = 0.9$  and  $k = 0.2$  (for the torispherical bottom), and  $h = 53"$  and multiply the volume found by  $72/96$ . In Case 2, calculate vertical cylindrical tank volumes with  $D = 72"$ ,  $a = 34"$  (for conical and spherical bottoms),  $f = 0.9$  and  $k = 0.2$  (for the torispherical bottom), and  $h = 53"$  and multiply the volume found by  $96/72$ . The results are summarized in the following table:

<u>Case</u>	Conical bottom (Gal)	Spherical bottom (Gal)	Torispherical bottom (Gal)
1	712.86	912.84	1,059.54
2	712.86	964.81	1,106.04

Calculated values for 'a' in the torispherical-bottom cases are 25.684" and 22.554" for Cases 1 and 2, respectively.

I acknowledge the invaluable help of Dr. Jimmie Lawson, Boyd Professor of Mathematics, and Mr. Hong Yin, both in the Department of Mathematics, Louisiana State University, Baton Rouge, LA. They were able to perform some of the symbolic integrations in the equations.

*Jones is a senior process chemist for Stockhausen Louisiana, LLC, Garyville, LA. Contact him at Dan.Jones@degussa.com*